1 A Model of Financial Markets with Quantitative and Discretionary Investors

In this section I propose a model of an equilibrium asset market in which quantitative investors ("quants") and discretionary investors ("discretionaries") interact. The model builds on Kacperczyk et al. (2016). Theirs is a static general equilibrium model with multiple assets subject to a common aggregate shock and to idiosyncratic shocks. Assets are traded by skilled investors, unskilled investors, and noise traders. Skilled investors can learn about assets' payoffs, but their learning capacity is limited. I augment that model along two dimensions: (1) I add a second group of skilled investors, quants, endowed with an unlimited learning capacity but able to learn only about idiosyncratic shocks. (2) I assume that private signals contain an unlearnable component (i.e. residual noise) that is heterogeneous across assets and investor types (i.e. quants or discretionaries).

1.1 Model

The model has three dates, $t = 1, 2, 3$. At $t = 1$, investors allocate their learning capacity. At $t = 2$, investors choose their portfolio allocations. At $t = 3$, prices and returns are realized.

1.1.1 *Assets and Risk Factors*

There are n risky assets and one riskless asset, with price 1 and payoff r. Of the risky ones, $n - 1$ are exposed to both idiosyncratic and aggregate risks, while n is a composite asset subject to the aggregate risk only. At t=3 the normally distributed risky assets' payoffs are:

$$f_i = \mu_i + b_i z_n + z_i \; and \; f_n = \mu_n + z_n. \tag{1}$$

where f_i is the payoff from asset i, for $i = 1, \ldots, n$; risk factors are given by $z = [z_1, \ldots, z_n]' \sim N(0, \Sigma)$; z_n represents the aggregate shock and z_i for $i \neq n$ idiosyncratic shocks. Σ is a diagonal matrix s.t. $\Sigma_{ii} = \sigma_i \in \mathbb{R}_+$; b_i is asset i's exposure to the aggregate risk, $\mu_i \in \mathbb{R}$ is its expected payoff. Rewriting system 1 in matrix form yields $f = \mu + \Gamma z$. The model is solved in terms of "synthetic" payoffs, affected by only one risk factor each: $\tilde{f} = \Gamma^{-1} f = \Gamma^{-1}\mu + z$.

The supply of the i^{th} risk factor is $\bar{x}_i + x_i$, where $x_i \sim N(0, \sigma_x)$ and $\bar{x}_i$ is an expected component. Supply is stochastic for prices not to be fully revealing. The aggregate risk factor is assumed to be the one in greatest supply since it affects all risky assets ($\bar{x}_n \gg \bar{x}_i \; \forall i \neq n$).

1.1.2 *Investors and Learning*

There is a unit mass of mean-variance investors with risk aversion ρ, indexed by $j \in [0, 1]$, of whom a fraction $\chi \in [0, 1]$ are skilled, the rest being unskilled. Among skilled investors, a fraction $\theta \in [0, 1]$ are quantitative and $(1 - \theta)$ are discretionary. Thus the measures of quants and discretionaries are, respectively, $\chi\theta$ and $\chi(1 - \theta)$.

Investors receive private signals about risk factors. Signal precision increases with the capacity for learning. Investor j's private signals vector is $s_j = [s_{1j}, \ldots, s_{nj}]'$, such that:

$$s_{ij} = z_i + \epsilon_{ij}, \; where: \; \epsilon_{ij} \sim N(0, \sigma_{ij}) \; for \; \sigma_{ij} \in [\underline{\sigma}_{ij}, \infty] \tag{2}$$

An infinite learning capacity allows reducing signals volatility to their lower-bound $\underline{\sigma}_{ij}$; whereas zero learning capacity leads to an infinite volatility ($\sigma_{ij} = \infty$).

The lower-bounds in signal volatility ensure that no investor, even those with unlimited learning capacity, can know the future with certainty. Their heterogeneity across assets captures potential differences in information availability. For instance, less information might be available about younger or smaller firms, hence a lower signal precision might be attainable. Lower-bounds also vary by investor type, the rationale being that information available for human consumption might not be machine-readable and vice-versa. Hence, the maximum precision achievable by quants or discretionaries might differ.

Discretionaries ($j = d \in [0, \chi(1-\theta)]$) have a learning capacity K, which they can allocate freely across all risk factors; so that the sum of their signals' precision is bounded:

$$\sum_{i=1}^{n} \sigma_{id}^{-1} = K \; where \; \sigma_{id}^{-1} \geq 0 \; \forall i = 1, \ldots n \tag{3}$$

Quants ($j = q \in [0, \chi\theta]$) have unlimited learning capacity about idiosyncratic risk factors $\forall\ i \neq n$; but do not receive a private signal about the aggregate shock $\sigma_{nq} = \infty$.

Unskilled investors ($j = u \in [0, 1 - \chi]$) have zero learning capacity hence $\sigma_{ij} = \infty\ \forall i = 1, \dots n$; which is equivalent to saying that they do not receive any private signal.

All investors learn from prices through the signals vector $s_p = [s_{1p}, \dots, s_{np}]'$, where:

$$s_i^p = z_i + \epsilon_{ip},\ where\ \epsilon_{ip} \sim N(0, \sigma_p). \tag{4}$$

1.1.3 *Recessions*

I derive differential predictions for expansions and recessions, modeled as periods of higher aggregate shock volatility, σ_n.[1].

1.2 Analysis

1.2.1 *Optimal Portfolio Choice*

In period $t = 2$, each investor j ($j = u, d, q$), given initial wealth W_0 and having risk aversion ρ, chooses the optimal portfolio allocation $\tilde{q_j}^*$ to maximize mean-variance utility:

$$\max_{\tilde{q_j}} U_{2j} = \left\{ \rho E_j[W_j] - \frac{\rho}{2} V_j[W_j] \right\} \text{ s.t. } W_j = rW_0 + \tilde{q_j}'(\tilde{f} - \tilde{p}r). \tag{5}$$

It follows that

$$\tilde{q_j}^* = \frac{1}{\rho} \hat{\Sigma_j}^{-1} \left(E_j[\tilde{f}] - \tilde{p}r \right). \tag{6}$$

Optimal allocation to risky assets decreases with risk aversion ρ, but increases with posterior private signal precision $\hat{\Sigma_j}^{-1}$ and expected payoff $E_j[\tilde{f}]$; the last two measures being group dependent. Investors with more precise signals, allocate more capital to risky assets.

[1] For empirical evidence in support of this assumption see footnote ??.

1.2.2 *Market Clearing*

Given the optimal portfolio choices of the different investors, the next step is to clear the asset market by equating the aggregate demand to supply such that:

$$\int \tilde{q}_j^* \, dj = \bar{x} + x. \tag{7}$$

The solution to the integral in equation (7) depends on the average learning capacity, across investor types, toward the different risk factors:

$$\bar{K_i} = \int K_{ij} \, dj = \begin{cases} \chi\theta K_{iq} + \chi(1-\theta)K_{id}, & i = 1, ..., n-1; \\ \chi(1-\theta)K_{id}, & i = n; \end{cases} \tag{8}$$

where $K_{iq} = \int \sigma_{iq}^{-1} \partial q$ and $K_{id} = \int \sigma_{id}^{-1} \partial d$. I solve for an equilibrium price of the form $pr = (A + B + Cx)$, where (A, B, C) depend on the model's parameters.

1.2.3 *Investors' Learning Choice*

At $t = 1$, all skilled investors choose their optimal learning capacity allocation.

Quants, having unlimited learning capacity, learn all available and machine processable information about idiosyncratic shocks *s.t.* $\sigma_{iq}^{-1} = \underline{\sigma}_{iq}^{-1}$ and $K_{iq} = \underline{\sigma}_{iq}^{-1}$ $\forall i \neq n$.

Discretionaries, having a limited learning capacity, must optimize their allocation. This choice depends on investors' expectation, at $t = 1$, of the distribution of excess returns at $t = 2$. After some manipulation, their expected utility at $t = 1$ can be written as:

$$U_{1d} = \frac{1}{2} \sum_{i=1}^{n} (\sigma_{id}^{-1} \lambda_i) + \text{constant} \tag{9}$$

where λ_i, the marginal benefit of learning about risk factor i, is given by:

$$\lambda_i \equiv \bar{\sigma}_i \{1 + [\rho^2(\sigma_x + \bar{x_i}^2) + \bar{K_i}]\bar{\sigma}_i\}; \quad \bar{\sigma}_i = \int \left(\hat{\Sigma}_j\right)_{ii} dj = \left(\sigma_i^{-1} + \bar{K_i} + \frac{\bar{K_i}^2}{\rho^2 \sigma_x}\right)^{-1}. \tag{10}$$

According to equation (10), λ_i increases with expected supply $\bar{x}_i$ and prior volatility σ_i but decreases with average private information in the market, $\bar{K}_i$ (Appendix B). The latter result is due to a substitution effect: when many investors learn about a given shock, the benefit that each derives from that knowledge is reduced. Equation (8) shows that $\bar{K}_i$ is a function of the share χ of skilled investors and among those the fraction θ that are quantitative.

The learning problem of discretionaries is given by:

$$\begin{gathered}\max_{K_{1d}\dots K_{nd}} \tfrac{1}{2}\textstyle\sum_{i=1}^{n}(\sigma_{id}^{-1}\lambda_i) + \text{constant} \\ \text{s.t.} \textstyle\sum_{i=1}^{n}\sigma_{id}^{-1} \leqslant K, \\ \underline{\sigma}_{id}^{-1} \geq \sigma_{id}^{-1} \geq 0 \quad \forall i = 1, ..., n.\end{gathered} \tag{11}$$

The solution to this problem consists in allocating all learning capacity to the risk factor i^* (when $i^* = \text{argmax}_i\, \lambda_i$) or to a basket of risk factors l^* such that $\lambda_{l^*} \in I_M/l^*$ and $\lambda_{l^*} = \text{argmax}_i\, \lambda_i$ (Van Nieuwerburgh and Veldkamp (2010)). A basket of risky assets could achieve the same marginal benefit λ_{i^*}, due to the substitution effect previously illustrated.[2]

An important difference from KVV's approach is that here the average signal precision about each risk factor ($\bar{K}_i$) also depends on the learning advantage (or disadvantage) of quants (eq. 8) and on the ratio θ of quants to discretionaries among skilled investors. This point is key for obtaining the model's main predictions.

1.3 Predictions

1.3.1 *Optimal Learning*

KVV show that, in recessions, the marginal benefit of learning about the aggregate shock (λ_n) increases with σ_n and ρ, hence capacity constrained investors focus their attention towards the aggregate shock. In expansions, when σ_n is lower, capacity constrained investors shift their attention towards idiosyncratic shocks, focusing on those with the highest volatility (σ_i). Similarly to KVV, my model predicts that discretionaries, being capacity constrained, shift from learning about the

[2] The solution is obtained through waterfilling. Non-symmetric equilibria where each investors might choose a different attention allocation are possible due to the same substitution effect.

aggregate shock in recessions to learning about idiosyncratic shocks in expansions (Appendix C.1). Quants, instead, specialize in learning about idiosyncratic shocks. Additionally, my model predicts that in recessions discretionaries display higher timing ability and lower picking ability than quants. These predictions derive directly from investors' optimal learning choices.

In expansions two opposing forces are at play. On the one hand, given their overall greater capacity for learning about idiosyncratic shocks, quants should experience higher picking ability than discretionaries. This effect though is opposed by the cross-asset heterogeneity in signal precision of quants ($\underline{\sigma}_{iq}^{-1}$). If discretionaries were to focus their attention on shocks for which relatively more information suitable for human consumption was available than for machine consumption, they could achieve a higher signal precision than quants on those stocks and overall greater picking ability, given their more concentrated allocations (Prop. 2).

Proposition 1. *Discretionaries shift their attention to the aggregate shock in recession; quants specialize in learning about idiosyncratic shocks. In recessions, discretionaries display higher timing ability and lower picking ability than quants.*

1.3.2 *Holdings*

Investors optimally hold more of what they know better. This is evident from equation (6), which shows that the optimal portfolio allocation $\tilde{q}_j^*$ is proportional to posterior precision $\hat{\Sigma}_j^{-1}$. Quants learn about more shocks, hence have a more precise signal about more of the risky assets; this leads them to hold more of them.

Proposition 2. *Quants optimally hold a greater number of stocks than do discretionaries.*

For discretionaries, the incentive to learn about shock i decreases the higher the precision of quantitative private signals about it ($\underline{\sigma}_{iq}^{-1}$). That is because both the average information about that shock in the market ($\bar{K}_i$) and discretionaries' informational disadvantage relative to quants increase. To better explain this last concept I define the *information gap* (G_{id}) as the difference in private signal precision about shock i between quants and discretionary investor d:

$$\begin{aligned} G_{id} &\equiv \left(K_{iq} - \sigma_{id}^{-1}\right) \quad \forall i = 1, ..., n-1, \\ K_{iq} &= \underline{\sigma}_{iq}^{-1} \quad \forall i = 1, ..., n-1. \end{aligned} \tag{12}$$

G_{id} is positive for most assets, since quants have a greater capacity for learning. It might be negative when more information is available for human than for machine consumption. G_{id} is strictly increasing in the precision of private signals of quants about asset i ($\underline{\sigma}_{iq}^{-1}$), as σ_{id}^{-1} is weakly decreasing in $\underline{\sigma}_{iq}^{-1}$. As a result, discretionaries optimally focus their attention on shocks for which their information gap with respect to quants is smaller (Appendix C.2).

Proposition 3. *An increase in the private signal precision of quants ($\underline{\sigma}_{iq}^{-1}$) weakly reduces attention allocation of discretionaries to risk factors with a greater information gap.*

1.3.3 *Dispersion of Opinion*

The dispersion of opinion of a representative quant (discretionary) with respect to other quants (discretionaries) is given by:

$$E\left[\left(\tilde{q}_q - \bar{\tilde{q}}_q\right)\left(\tilde{q}_q - \bar{\tilde{q}}_q\right)^{'}\right] = \frac{1}{\rho^2}\sum_{i=1}^{n-1}\underline{\sigma}_{iq}^{-1} \tag{13}$$

$$E\left[\left(\tilde{q}_d - \bar{\tilde{q}}_d\right)\left(\tilde{q}_d - \bar{\tilde{q}}_d\right)^{'}\right] = \frac{1}{\rho^2}K + \frac{1}{\rho^2}\sum_{i=1}^{n}\left(\sigma_{id}^{-1} - K_{id}\right)^2\lambda_i \tag{14}$$

where $\bar{\tilde{q}}_j = \int\left[\frac{1}{\rho}\hat{\Sigma}_j^{-1}\left(E_j[\tilde{f}] - \tilde{p}r\right)\partial j\right]$ $for\ j \in [q, d]$ (Appendix C.3).

Dispersion of opinion is determined by two effects. First, it increases with the total precision of private signals: the greater the total precision of private signals the more weight is given to the heterogeneous private signals as opposed to common priors in determining posteriors. Second, dispersion of opinion increases with the cumulative difference in the attention allocated by investors to each asset with respect to the attention allocated to the same assets by the average investor of their type. Risk tolerance magnifies both effects.

For quants (eq. 13) dispersion of opinion is entirely determined by the first effect; attention allocation is always symmetric among quants as they optimally learn all available and machine processable information. For discretionaries (eq. 14) both effects are at play. The first term shows that dispersion of opinion is an increasing function of total private signal precision K. The second term shows that the greater the difference in signal precision of investor d with respect to other discretionaries $\left(\sigma_{id}^{-1} - K_{id}\right)$, the greater the dispersion of opinion. Discretionaries must optimally allocate their limited learning capacity; due to a substitution effect they might choose to learn about different shocks, increasing dispersion of opinion (Appendix C.3).

Proposition 4. *As long as* $\sum_{i=1}^{n} \left(\sigma_{id}^{-1} - K_{id}\right)^2 \lambda_i > \sum_{i=1}^{n-1} \underline{\sigma}_{iq}^{-1} - K$*, dispersion of opinion is greater among discretionaries than among quants.*

1.3.4 *Performance*

An investor's risk-adjusted performance is measured as his expected excess return with respect to the market return. The excess return of investor j is given by (Appendix A):

$$E\left[(R_j - R_M)\right] = E\left[\left(\tilde{q}_j^* - \bar{\tilde{q}}\right)' \left(\tilde{f} - \tilde{p}r\right)\right] = \frac{1}{\rho} \sum_{i=1}^{n} \left[\lambda_i \left(K_{ij} - \bar{K}_i\right)\right]. \tag{15}$$

Expected excess returns increase with the precision of the private signals of investor j on the risk factors he chooses to learn about ($\forall i \ s.t. \ K_{ij} > 0$) and in proportion to the marginal benefit λ_i of learning about them. They decrease with increases in the average precision of private signals about asset i across all investors in the market ($\bar{K}_i$).

Discretionary investor d, when learning about the aggregate shock, earns a positive excess return if:

$$\lambda_n \left[K - \chi(1-\theta)K_{nd}\right] > \chi\theta \sum_{i=1}^{n-1} \left[\lambda_i \underline{\sigma}_{iq}^{-1}\right]. \tag{16}$$

where the left-hand-side represents his informational advantage with respect to both unskilled investors and quants (i.e., his ability to learn about the aggregate shock when its volatility is high); and the right-hand-side represents his learning disadvantage with respect to quants (i.e., his overall lower capacity for learning). When, instead, he learns about idiosyncratic shocks,[3] his

[3]Without loss of generality I assume that each discretionary investor d learns about one idiosyncratic shock l.

expected excess return is positive if:

$$\lambda_l \left[K - \chi(1-\theta)K_{ld} - \chi\theta\underline{\sigma}_{lq}^{-1} \right] > \chi\theta \sum_{i\neq(j,n)} \left[\lambda_i \underline{\sigma}_{iq}^{-1} \right]. \tag{17}$$

This second condition is more restrictive for two reasons. First, he allocates the same amount of total capacity (K) to learning about a shock in much lower supply ($\bar{x}_n >> \bar{x}_l \; \forall l \neq n$). Second, both investor types learn about the same risk factors, implying a higher average precision of private signals ($\bar{K}_l$), and a lower marginal benefit of learning (eq. 21). Moreover, an increase in the prior volatility of the idiosyncratic shock l, which he chooses to learn about, increases his expected excess return only if θ is sufficiently low: $\theta < \frac{K-\chi K_{ld}}{\chi \bar{G}_l}$ (for $\bar{G}_l = \int G_{ld}\partial d$). Whereas when he optimally allocates all of his attention to learn about the aggregate shock, increases in σ_n always have a positive effect on his excess return. When he shifts his attention from idiosyncratic to the aggregate shock, an increase in σ_n always increases his performance if he held stocks with a low information gap. Finally, increases in σ_i, for $i \neq l \neq n$ always reduce his performance. These observations, taken together with equations (16) and (17), lead to conclude that discretionaries have higher expected excess returns in recessions.

From the above condition we further observe that, when discretionaries learn about shocks with a smaller average information gap, an increase in λ_l increases their excess return for a wider range of θ values – always rising if $\bar{G}_l < 0$. Discretionaries tend to allocate their attention towards risk factors with a smaller information gap (Prop. 3). This also affects their expected excess returns, which always decrease when the private signal precision of quants ($\underline{\sigma}_{iq}^{-1}$) rises for shocks that they pay attention to (Appendix C.4).

Proposition 5. *The performance of discretionaries weakly increases with σ_n (i.e. in recessions), and when learning about shocks with a low information gap.*

Quants do not reallocate attention. Hence, their expected excess returns depends only on changes in the marginal benefit of learning about the various shocks and the consequent attention reallocation by discretionaries.

This could also be a basket of shocks with the same and highest marginal benefit of learning.

When discretionaries learn about the aggregate shocks, quants' performance is positive if:

$$(1-\chi\theta)\sum_{i=1}^{n-1}\left[\lambda_i\underline{\sigma}_{iq}^{-1}\right] > \chi(1-\theta)\lambda_n K \tag{18}$$

When discretionaries learn about a basket of shocks $l \neq n$, with the same and highest marginal benefit of learning λ_l^*, the performance of quants is positive if:

$$(1-\chi\theta)\sum_{i=1}^{n-1}\left[\lambda_i\underline{\sigma}_{iq}^{-1}\right] > \chi(1-\theta)\sum_{l\neq n}\lambda_l^* K_{ld} = \chi(1-\theta)\lambda_l^* K \tag{19}$$

In comparing the above conditions, two mechanisms are at play: when discretionaries learn about the aggregate shock, quants are at a learning disadvantage with respect to the shock in greatest supply (n); making condition 18 more restrictive. When quants and discretionaries learn about the same shocks, the average private signal precision about those shocks increases, decreasing their marginal benefit of learning; making condition 19 more restrictive. A similar reasoning applies when looking at the effect of an increase in the volatility of the aggregate shock σ_n on the performance of quants. The direct effect is a reduction in performance through an increase in λ_n. The indirect effect is a weak increase in performance through attention reallocation; i.e. discretionaries reduce the attention allocated to the basket or shocks $l \neq n$, increasing its marginal benefit of learning. Hence, we cannot unequivocally say whether the performance of quants should increase or decrease in recessions; but an increase in σ_n always has a larger positive impact for discretionaries.

Further, the performance of quants always increases with the volatility of shocks that discretionaries don't learn about (σ_i for $i \neq l \neq n$); i.e. with increases in the marginal benefit of learning of shocks for which they have a learning advantage. Quantitative performance increases with the volatility of the shocks that discretionaries also learn about (σ_l) only when the information gap is sufficiently high. The performance of quants, though, is not always increasing in the information gap. The information gap strictly increases with the precision of private signals of quants $\underline{\sigma}_{iq}^{-1}$. This has two effects on their performance: it directly increases excess returns thanks to their greater informativeness in choosing portfolio allocations; it indirectly decreases excess returns as the higher average precision of private signals about those shocks reduces their marginal benefit of learning. An increase in $\underline{\sigma}_{iq}^{-1}$ only increases the performance of quants if their total share in the market ($\chi\theta$) is sufficiently low (eq. 25).

Finally, quantitative performance worsens as the share of quants among skilled investors (θ) increases. The negative effect of an increase in θ on excess returns is greater when the total share of quants ($\chi\theta$) is high and their private signal precision ($\underline{\sigma}_{iq}^{-1}$) rises. For a high enough $\chi\theta$ an increase in $\underline{\sigma}_{iq}^{-1}$ causes excess returns to decrease faster as θ increases (eq. 27). This is particularly relevant as quants have unconstrained learning capacity and they all learn about the same shocks. Hence, a small increase in θ can greatly impact λ_is (Appendix C.5).

Proposition 6. *When σ_n rises (i.e. in recessions), quants experience a smaller increase in performance than discretionaries. Their performance always decreases as the share of skilled investors who are quantitative rises; particularly when their overall signal precision and the total share of quants in the market is high.*

APPENDIX

A Model Solution

The key steps of the model's solution are summarized below. For more details see Admati (1985) and Kacperczyk et al. (2016). The key difference in my model is that the obtained solution is a function of the fraction of skilled investors who are quantitative and of their differential signal precision. This is key in deriving the model's predictions.

A.1 *Market-clearing*

The model is solved in terms of synthetic assets, only affected by one risk-factor each. Solving for the market clearing condition we obtain:

$$A = \Gamma^{-1}\mu - \rho\bar{\Sigma}\bar{x};\ B = I - \bar{\Sigma}\Sigma^{-1};\ C = -\rho\bar{\Sigma}\left(I - \frac{1}{\rho^2\sigma_x}\bar{\Sigma}_\eta^{-1}\right),$$

$$\tilde{p}r = \Gamma^{-1}\mu + \bar{\Sigma}\left[\left(\bar{\Sigma}^{-1} - \Sigma^{-1}\right)z - \rho(\bar{x} - x) - \frac{1}{\rho^2\sigma_x}\bar{\Sigma}_\eta^{-1}x\right]$$

Where: $\bar{\Sigma}^{-1} = \Sigma^{-1} + \Sigma_p^{-1} + \bar{\Sigma}_\eta^{-1};\ \Sigma_P^{-1} = \left(\sigma_x B^{-1}CC'B^{-1'}\right)^{-1} = \frac{1}{\rho^2\sigma_x}\bar{\Sigma}_\eta^{-1'}\bar{\Sigma}_\eta^{-1}$

A.2 *Discretionary learning choice*

Discretionaries solve the below problem to determine their optimal learning:

$$\begin{cases} U_{1d} = E_1\left[\rho E_d\left[W_d\right] - \frac{\rho^2}{2}V_d\left[W_d\right]\right] \\ W_d = rW_0 + \tilde{q}_d^{*'}\left(\tilde{f} - \tilde{p}r\right) \\ \tilde{q_d}^* = \frac{1}{\rho}\hat{\Sigma}_d^{-1}\left(E_d[\tilde{f}] - \tilde{p}r\right) \end{cases}$$

The solution is obtained by: (1) Substituting $\tilde{q_d}^*$ into W_d. (2) Computing $E_d\left[W_d|\hat{E}_d\left[\tilde{f}\right], \hat{\Sigma}_d\right]$ and $V_d\left[W_d|\hat{E}_d\left[\tilde{f}\right], \hat{\Sigma}_d\right]$. (3) Substituting the derived moments into the equation for U_{1d}.

A.3 *Expected excess returns*

The expected excess return is obtained by: (1) solving for $(\tilde{f} - \tilde{p}r)$. (2) solving for $(\tilde{q}_j - \tilde{\bar{q}})$. (3) substituting them into: $(q_j - \bar{q})'(f - pr) = (q_j - \bar{q})'\Gamma^{-1}(\Gamma f - \Gamma pr) = (\tilde{q}_j - \tilde{\bar{q}})'(\tilde{f} - \tilde{p}r)$. (4) taking the expectation: $E[(\tilde{q}_j - \tilde{\bar{q}})'(\tilde{f} - \tilde{p}r)] = \rho Tr(\bar{x}'\bar{\Sigma}\Delta\bar{\Sigma}\bar{x}) + \frac{1}{\rho}Tr(\Delta V) = \frac{1}{\rho}\sum_{i=1}^{n}\left[\lambda_i\left(K_{ij} - \bar{K}_i\right)\right]$.

B Comparative Statics

Derivations below are simplified using: $(\lambda_i - \bar{\sigma}_i) = [\rho^2(\sigma_x + \bar{x}_i{}^2) + \bar{K}_i]\bar{\sigma}_i^2 \equiv \gamma_i\bar{\sigma}_i^2 > 0$ (eq. 10).
The sensitivity of the marginal benefit of learning about risk factor i with respect to its expected supply is positive and is given by:

$$\frac{\partial\lambda_i}{\partial\bar{x}_i^2} = \frac{\partial}{\partial\bar{x}_i^2}\left[\bar{\sigma}_i + \bar{\sigma}_i{}^2\rho^2\sigma_x + \bar{\sigma}_i{}^2\rho^2\sigma_x\bar{x}_i^2 + \bar{\sigma}_i{}^2\bar{K}_i\right] = \bar{\sigma}_i{}^2\rho^2 > 0 \tag{20}$$

The sensitivity of the marginal benefit of learning about risk factor i with respect to the average precision of private signals about i $(\bar{K}_i)$ is negative and is given by:

$$\frac{\partial\lambda_i}{\partial\bar{K}_i} = -2\left(\frac{\bar{K}_i}{\rho^2\sigma_x}\left(2\lambda_i - \bar{\sigma}_i\right) + \lambda_i - \bar{\sigma}_i\right) < 0 \tag{21}$$

The sensitivity of the marginal benefit of learning about risk factor i with respect to its volatility is positive and is given by:

$$\frac{\partial\lambda_i}{\partial\sigma_i} = \left(\frac{\bar{\sigma}_i}{\sigma_i}\right)^2 + 2\left(\frac{\lambda_i - \bar{\sigma}_i}{\sigma_i}\right) > 0 \tag{22}$$

This positive effect is decreasing (resp. increasing) in the share of skilled investors that are quantitative θ for $i = 1, ..., n-1$, for shocks with a positive (resp. negative) average information gap $(\bar{G}_{id} = (\underline{\sigma}_{iq}^{-1} - \bar{K}_{id}))$. It is always increasing for $i = n$, as $\underline{\sigma}_{nq}^{-1} = 0$:

$$\frac{\partial\lambda_i}{\partial\sigma_i\partial\theta} = -2\chi\left(\underline{\sigma}_{iq}^{-1} - \bar{K}_{id}\right)\left\{3\left(\lambda_i - \bar{\sigma}_i\right)\left(\frac{2\bar{K}_i}{\rho^2\sigma_x} + 1\right) + \bar{\sigma}_i\left(\frac{2\bar{K}_i}{\rho^2\sigma_x}\right)\right\}\left(\frac{\bar{\sigma}_i}{\sigma_i}\right)^2 \tag{23}$$

The sensitivity of the marginal benefit of learning about risk factor i with respect to the average precision of private signals of quants $(\underline{\sigma}_{iq}^{-1})$ is negative and given by:

$$\frac{\partial\lambda_i}{\partial\underline{\sigma}_{iq}^{-1}} = -2\chi\theta\bar{\sigma}_i\left(\frac{\bar{K}_i}{\rho^2\sigma_x}\left(2\lambda_i - \bar{\sigma}_i\right) + \lambda_i - \bar{\sigma}_i\right) < 0 \tag{24}$$

The sensitivity of the total benefit of learning about risk factor i for quants with respect to the average precision of their private signals ($\underline{\sigma}_{iq}^{-1}$) is given by:

$$\frac{\partial \lambda_i \underline{\sigma}_{iq}^{-1}}{\partial \underline{\sigma}_{iq}^{-1}} = \lambda_i - 2\chi\theta\bar{\sigma}_i \left(\frac{\bar{K}_i}{\rho^2\sigma_x} \left(2\lambda_i - \bar{\sigma}_i\right) + \lambda_i - \bar{\sigma}_i \right) \underline{\sigma}_{iq}^{-1} \tag{25}$$

which is positive iff: $\chi\theta < \frac{\lambda_i}{2\bar{\sigma}_i \left(\frac{\bar{K}_i}{\rho^2\sigma_x}(2\lambda_i - \bar{\sigma}_i) + \lambda_i - \bar{\sigma}_i \right) \underline{\sigma}_{iq}^{-1}}$.

The sensitivity of the marginal benefit of learning about risk factor i with respect to the share of skilled investors using quantitative strategies (θ) is given by:

$$\frac{\partial \lambda_i}{\partial \theta} = -2\chi \left(\underline{\sigma}_{iq}^{-1} - \bar{K}_{id}\right) \bar{\sigma}_i \left(\frac{\bar{K}_i}{\rho^2\sigma_x} \left(2\lambda_i - \bar{\sigma}_i\right) + \lambda_i - \bar{\sigma}_i \right) \tag{26}$$

which is negative $\forall i \neq n$ for which the average information gap $\left(\underline{\sigma}_{iq}^{-1} - \bar{K}_{id}\right)$ is positive; it is positive otherwise (i.e. for $i = n$ or for $i \neq n$ and $\left(\underline{\sigma}_{iq}^{-1} - K_{id}\right) < 0$). The partial derivative of $\frac{\partial \lambda_i}{\partial \theta}$ with respect to $\underline{\sigma}_{iq}^{-1}$ is equal to 0 for $i = n$ (since $\underline{\sigma}_{iq}^{-1} = 0$), it is otherwise quadratic with respect to the total share of quants ($\chi\theta$) such that:

$$\begin{aligned}
\frac{\partial \lambda_i}{\partial \theta \partial \underline{\sigma}_{iq}^{-1}} &= a \left(\underline{\sigma}_{iq}^{-1} - \bar{K}_{id}\right) \chi^2\theta^2 + b \left(\underline{\sigma}_{iq}^{-1} - \bar{K}_{id}\right) \chi\theta + c \\
a &= 2\chi\bar{\sigma}_i^2 \frac{1}{\rho^2\sigma_x} \left(\bar{\sigma}_i + 2\bar{\sigma}_i^2\gamma_i\right) > 0 \\
b = 2\chi\bar{\sigma}_i^2 \left(\frac{2\bar{K}_i}{\rho^2\sigma_x} + 1 \right) &\left[\frac{2\bar{K}_i}{\rho^2\sigma_x} \left(1 + \bar{\sigma}_i^2\gamma_i + 2\bar{\sigma}_i\gamma_i\right) + 2\bar{\sigma}_i\gamma_i + \bar{\sigma}_i^2\gamma_i - \bar{\sigma}_i \right] > 0 \\
c &= -2\chi\bar{\sigma}_i^2 \left[\left(\frac{2\bar{K}_i}{\rho^2\sigma_x} + 1 \right) \left(\bar{\sigma}_i\gamma_i + 1\right) + \frac{\bar{K}_i}{\rho^2\sigma_x} + 1 \right] < 0
\end{aligned} \tag{27}$$

which is always negative for a negative average information gap – i.e. an increase in $\underline{\sigma}_{iq}^{-1}$ determines a slower decrease in λ_i corresponding to an increase in θ. When the average information gap is positive the effect depends on the level of $\chi\theta \in [0, 1]$: for ($\chi\theta = 0$) it is negative, for ($\chi\theta = 1$) it is positive. Hence there exists a $(\chi\theta)^*$ above which an increase in $\underline{\sigma}_{iq}^{-1}$ determines a faster decrease in λ_i corresponding to an increase in θ.

C Proofs

C.1 *Proposition 1*

Constrained discretionaries, adapt their learning depending on changes in the marginal benefit of learning about each shock (λ_i), which increases with shock i's volatility, σ_i (eq. 22) and with its expected supply, $\bar{x}_i^2$ (eq. 20). Kacperczyk et al. (2016), show through waterfilling that increases in σ_i weakly increase the attention allocated to shock i by capacity constrained investors (attention reallocation is unchanged when no attention or all attention was already allocated to shock i prior to the increase in σ_i). They further show that this effect is magnified for the aggregate shock in recessions when both σ_i and risk-aversion ρ increase: increases in ρ increase the marginal benefit of learning about assets in greater supply, the aggregate shock is in greatest supply. In my model this result in unchanged. Indeed, theirs is a special case of my model, when the share of quants (θ) is zero. Increases in the θ further increase the benefit of learning about the aggregate shock when σ_n increases (eq. 23). Hence, in recessions, discretionaries in my model behave like all skilled investors in KVV and focus on learning about the aggregate shock. Quants, instead, not being capacity constrained, optimally learn all available and machine processable information about idiosyncratic shocks, while they don't acquire private signals about the aggregate shock. Hence, their attention allocation is not affected by increases in σ_n or ρ.

C.2 *Proposition 3*

An increase in private signal precision of quants about shock l ($\underline{\sigma}_{lq}^{-1}$) decreases the marginal benefit of learning about it (eq. 24). An increase in the share of quants (θ) decrease the marginal benefit of learning about shocks with higher $\underline{\sigma}_{lq}^{-1}$ (eq. 26). In order to verify how this affects the attention allocated by discretionary investor d to shock l, we need to consider how attention was allocated before the increase. CASE (1): If before the increase in $\underline{\sigma}_{lq}^{-1}$ no attention was allocated to shock l no attention reallocation happens: an increase in $\underline{\sigma}_{lq}^{-1}$ reduces the marginal benefit of learning about shock l. An increase in θ also reduces the marginal benefit of learning about shock l if this has a positive information gap. If the information gap is negative, an increase in θ increases λ_l

but, being λ_l continuous in θ, a marginal change in θ cannot change the discrete ranking of λs. CASE (2): If before the increase in $\underline{\sigma}_{lq}^{-1}$ or in θ all attention was allocated to λ_l, then no attention reallocation happens: an increase in $\underline{\sigma}_{lq}^{-1}$ always decrease λ_l; an increase in θ decreases λ_l when the information gap is positive; but, being λ_l continuous in $\underline{\sigma}_{lq}^{-1}$ and θ, a marginal increase in $\underline{\sigma}_{lq}^{-1}$ or θ cannot change the discrete ranking of $\lambda_i s$. For a negative information gap, an increases in θ increase λ_l further, so there is no incentive to reallocate attention. CASE (3): If before the increase in $\underline{\sigma}_{lq}^{-1}$ or in θ, l was within the basket of assets among which attention was allocated, there is attention reallocation. The new equilibrium is obtained by waterfilling (Cover and Thomas (1991); Kacperczyk et al. (2016)). Case 3a: increases in $\underline{\sigma}_{lq}^{-1}$ decrease λ_l, decreasing the attention allocated to shock l. This, in turn, increases the incentive to learn about it, due to substitution effects ($\frac{\partial \lambda_i}{\partial K_l} < 0$; eq. 21). Through waterfilling, I reallocate attention among shocks in the basket until either no attention is allocated to shock l or the marginal benefit of learning about all shocks is equalized. I construct a new equilibrium. Consider the set $I \ s.t. \ \lambda_{l'} = argmax_{\lambda_i} \ \forall l' \in I$. Then an increase in $\underline{\sigma}_{l''q}^{-1}$ for $l'' \in I$ leads to a decrease in $\lambda_{l''}$. To restore the equilibrium I reallocate attention from l'' to l', $\forall l' \in I$ until $\lambda_{l'} = \lambda_{l''} \ \forall l', \ l'' \in I$. In the new equilibrium both $\lambda_{l'}$ and $\sigma_{dl''}^{-1}$ are smaller. Case 3b: for $G_l > 0$ (resp.$G_l < 0$), an increase in θ decreases (resp. increases) λ_l, this decreases (resp. increases) the attention allocated to shock l (σ_{dl}^{-1}). This, in turn, increases (resp. decreases) the incentive to learn about shock l, due to substitution effects. Through waterfilling, I reallocate attention among the shocks in the basket until either no (resp. all) attention is allocated to shock l or the marginal benefit of learning about all shocks is equalized. An increase in θ has a negative (resp. positive) effect on *all* shocks in the basket; $\frac{\partial \lambda_i}{\partial \theta}$, though, is strictly decreasing in G_l. This implies that an increase in θ will have a larger effect in decreasing (resp. increasing) the marginal benefit of learning about those shocks with a greater (resp. smaller) G_l. Therefore, there exists $G_l^* \ s.t. \ \frac{\partial \lambda_l}{\partial \theta} < \frac{\partial \lambda_l'}{\partial \theta} \forall l'$ (resp. $\frac{\partial \lambda_l}{\partial \theta} > \frac{\partial \lambda_l'}{\partial \theta} \forall l'$) if $G_l < G_l^*$; so attention is shifted to those shocks in the basket with a smaller information gap. To summarize, if investor d was allocating attention to shock l, after an increase in $\underline{\sigma}_{lq}^{-1}$ he decreases attention to it. Hence, an increase in $\underline{\sigma}_{lq}^{-1}$ weakly decreases σ_{id}^{-1} and strictly increases G_{ld} (eq: 12). Finally, an increase in θ shifts attention to shocks with a lower G_{ld}. Note that, in the new equilibrium, the attention allocated to each shock in basket

l by the different discretionary investors might differ. That is due to the same substitution effect. The optimal marginal benefit of learning λ_l^* will be higher the lower the commonality in attention allocation among discretionaries.

C.3 *Proposition 4*

Let us define the risk factor portfolio of the average investor as:

$$\int \tilde{q}_j^* \, dj = \frac{1}{\rho} \int \hat{\Sigma}_j^{-1} \left(E_j[\tilde{f}] - \tilde{p}r \right) \partial j = \frac{1}{\rho} \int \left[\hat{\Sigma}_j^{-1} \left(\Gamma' \mu + E_j[z] \right) \right] dj - \bar{\Sigma}_j^{-1} \tilde{p}r$$

$$= \frac{1}{\rho} \left\{ \int \left[\hat{\Sigma}_j^{-1} E_j[z] \right] dj + \bar{\Sigma}^{-1} \left(\Gamma' \mu - \tilde{p}r \right) \right\} = \frac{1}{\rho} \left\{ \int \left[\hat{\Sigma}_j^{-1} \hat{\Sigma}_j \left(\Sigma_{\eta j}^{-1} \eta_j + \Sigma_p^{-1} \eta_p \right) \right] dj + \bar{\Sigma}_j^{-1} \left(\Gamma' \mu - \tilde{p}r \right) \right\}$$

$$\int \tilde{q}_j^* \, \partial j = \frac{1}{\rho} \left[\Sigma_{\eta j}^{-1} z + \Sigma_p^{-1} \eta_p + \bar{\Sigma}_j^{-1} \left(\Gamma' \mu - \tilde{p}r \right) \right] \tag{28}$$

Then, the risk factor portfolios for the average quant and discretionary are:

$$\bar{\tilde{q}}_q = \frac{1}{\rho} \left[\Sigma_{\eta q}^{-1} z - \Sigma_p^{-1} \eta_p - \hat{\Sigma}_q^{-1} \left(\Gamma' \mu - \tilde{p}r \right) \right]; \quad \bar{\tilde{q}}_d = \frac{1}{\rho} \left[\bar{\Sigma}_{\eta d}^{-1} z - \Sigma_p^{-1} \eta_p - \bar{\Sigma}_d^{-1} \left(\Gamma' \mu - \tilde{p}r \right) \right] \tag{29}$$

where $\bar{\Sigma}_d^{-1} = \Sigma^{-1} + \Sigma_p^{-1} + \int \Sigma_{\eta d}^{-1} \partial d$; $\bar{\Sigma}_q^{-1} = \Sigma^{-1} + \Sigma_p^{-1} + \int \Sigma_{\eta q}^{-1} \partial q = \Sigma^{-1} + \Sigma_p^{-1} + \Sigma_{\eta q}^{-1} = \hat{\Sigma}_q^{-1}$.

The last equality indicates that quants optimally drive the volatility of their private signals to their lower-bounds. Then, the differences between the risky portfolio of a quantitative or discretionary investor and that of the average investor of the same type are:

$$(\tilde{q}_q^* - \bar{\tilde{q}}_q) = \frac{1}{\rho} \left[\hat{\Sigma}_q^{-1} \left(E_q[\tilde{f}] - \tilde{p}r \right) - \Sigma_{\eta q}^{-1} z + \Sigma_p^{-1} \eta_p + \hat{\Sigma}_q^{-1} \left(\Gamma' \mu - \tilde{p}r \right) \right] = \frac{1}{\rho} \Sigma_{\eta q}^{-1} \epsilon_q \tag{30}$$

$$\begin{aligned} (\tilde{q}_d^* - \bar{\tilde{q}}_d) &= \frac{1}{\rho} \left[\hat{\Sigma}_d^{-1} \left(E_d[\tilde{f}] - \tilde{p}r \right) - \bar{\Sigma}_{\eta d}^{-1} z + \Sigma_p^{-1} \eta_p + \bar{\Sigma}_d^{-1} \left(\Gamma' \mu - \tilde{p}r \right) \right] = \\ &= \frac{1}{\rho} \left[\left(\Sigma_{\eta d}^{-1} - \bar{\Sigma}_{\eta d}^{-1} \right) \left(\tilde{f} - \tilde{p}r \right) + \Sigma_{\eta d}^{-1} \epsilon_d \right] \end{aligned} \tag{31}$$

where: $\hat{\Sigma}_j^{-1} \left(E_j[\tilde{f}] - \tilde{p}r \right) = \hat{\Sigma}_j^{-1} \left(\Gamma' \mu + \hat{\Sigma}_j \left(\Sigma_{\eta j}^{-1} \eta_j + \Sigma_p^{-1} \eta_p \right) - \tilde{p}r \right)$.

Eq. 31 contains one more term than eq. 30, as discretionaries don't all necessarily have the same precision of private signals. Indeed, in allocating their limited capacity, they might optimally choose to learn about different shocks, to limit the average precision of private signals about any given shock.

Finally, dispersion of opinion is defined as the expected square difference between the risky portfolio of an investor and that of the average investor of his type:

$$E\left[\left(\tilde{q}_q^* - \bar{\tilde{q}}_q\right)\left(\tilde{q}_q^* - \bar{\tilde{q}}_q\right)'\right] = \frac{1}{\rho^2}\sum_{i=1}^{n-1}\underline{\sigma}_{iq}^{-1} \tag{32}$$

$$\begin{aligned} E\left[\left(\tilde{q}_d^* - \bar{\tilde{q}}_d\right)\left(\tilde{q}_d^* - \bar{\tilde{q}}_d\right)'\right] &= \frac{1}{\rho^2}E\left[\sum_{i=1}^{n}\left[\left(\sigma_{id}^{-1} - K_{id}\right)\left(\tilde{f} - \tilde{p}r\right) + \sigma_{id}^{-1}\epsilon_{id}\right]^2\right] = \\ &= \frac{1}{\rho^2}\left[\sum_{i=1}^{n}\left(\sigma_{id}^{-1} - K_{id}\right)^2\lambda_i\right] + \frac{1}{\rho^2}K \end{aligned} \tag{33}$$

Dispersion of opinion is greater among discretionaries if the portfolio dispersion among discretionaries more than compensates for their capacity disadvantage with respect to quants: $\sum_{i=1}^{n}\left(\sigma_{id}^{-1} - K_{id}\right)^2\lambda_i > \sum_{i=1}^{n-1}\underline{\sigma}_{iq}^{-1} - K$.

C.4 *Proposition 5*

The expected excess return of discretionaries is given by:

$$E[R_D - R_M] = \frac{1}{\rho}\sum_{l}\lambda_l\left[\sigma_{ld}^{-1} - \chi(1-\theta)K_{ld} - \underline{\sigma}_{lq}^{-1}\chi\theta\right] + \frac{1}{\rho}\lambda_n\left[\sigma_{nd}^{-1} - \chi(1-\theta)K_{nd}\right] - \frac{1}{\rho}\chi\theta\sum_{i\neq l}^{n-1}\lambda_i\underline{\sigma}_{iq}^{-1}$$

l is a basket of idiosyncratic shocks that discretionaries might pay attention to.

Increases in $\boldsymbol{\sigma_{i'}}$ *for* $\boldsymbol{i' \in [i, l, n]}$: An increase in $\sigma_{i'}$ directly lowers $\lambda_{i'}$. The overall effect on performance depends on whether the increase causes any attention reallocation.

σ_n rises: There are three possible scenarios. CASES (1) and (2): If discretionary investor d did not pay attention to the aggregate shock or if he allocated all his attention to it prior to the increase in σ_n, no attention reallocation happens (sec. C.1). When all of d's attention was allocated to the aggregate shock, his expected excess return strictly increases due to an increase in λ_n, it is otherwise unchanged. CASE (3): Discretionary investor d allocated some attention to the aggregate shock as part of a basket of shocks prior to the increase in σ_n. An increase in λ_n either causes all attention to be shifted away from the other shocks in the basket and towards the aggregate shock, or it causes some attention to be reallocated to the aggregate shock until the marginal benefit of learning for all shocks is equalized. In both cases the attention paid to the aggregate shock,

K_{nd}, and its marginal benefit of learning, λ_n, increase (prop 1). The shift of attention away from other shocks in the basket, though, raises their marginal benefit of learning (λ_l). This decreases performance by increasing the overall informational advantage of quants. Case (3a): The net effect on performance is always positive when, after the increase, discretionaries still learn about the same basket of shocks. In that case the benefit of learning about all shocks in the basket (λ_l^A) is strictly higher than the marginal benefit for those same shocks before the increase (λ_l^B), while the learning capacity of quants and the marginal benefit of learning about other shocks is unchanged. A sufficient condition for performance to be positive is:

$$\begin{aligned} \lambda_l^A \left[(\sigma_{ld}^{-1})^A - \chi(1-\theta)K_{ld}^A - \underline{\sigma}_{lq}^{-1}\chi\theta + (\sigma_{nd}^{-1})^A - \chi(1-\theta)K_{nd}^A\right] > \\ \lambda_l^B \left[(\sigma_{ld}^{-1})^B - \chi(1-\theta)K_{ld}^B - \underline{\sigma}_{lq}^{-1}\chi\theta + (\sigma_{nd}^{-1})^B - \chi(1-\theta)K_{nd}^B\right] \end{aligned} \tag{34}$$

This is always verified since $\lambda_l^A > \lambda_l^B$ and from equation 3 we know that:
$\left[(\sigma_{ld}^{-1})^A + (\sigma_{nd}^{-1})^A\right] = \left[(\sigma_{ld}^{-1})^B + (\sigma_{nd}^{-1})^B\right] = K$ and $\left[K_{ld}^A + K_{nd}^A\right] = \left[K_{ld}^B + K_{nd}^B\right] = K$.
Case 3b: When all attention is shifted to the aggregate shock, $\lambda_n^A > \lambda_l^A > \lambda_l^B = \lambda_n^B = \lambda^B$. Here a sufficient condition for the expected excess return to be positive is:

$$(\lambda_n^A - \lambda^B)K\left[1-\chi(1-\theta)\right] > \chi\theta(\lambda_l^A - \lambda^B)\underline{\sigma}_{lq}^{-1}; \tag{35}$$

always satisfied if $\chi < \frac{K}{\underline{\sigma}_{lq}^{-1}}\frac{(\lambda_n^A-\lambda^B)}{(\lambda_l^A-\lambda^B)}$. Discretionaries learn about shocks with low information gap, G_l (prop. 3). The smaller $\underline{\sigma}_{lq}^{-1}$, the higher the maximum χ for which the condition is satisfied (always satisfied for $\underline{\sigma}_{lq}^{-1} - K < 0$). So, as long as discretionaries learn about low G_l shocks, an increase in σ_n weakly increases their expected excess return.

$\sigma_{i\neq n\neq j}$ *rises*: An increase in the volatility of shocks that discretionary investor d does not learn about strictly decreases his expected excess return. That is because a marginal change in σ_i cannot change the discrete ranking of λs, hence no attention is reallocated; while the competitive advantage of quants with respect to shock i increases.

σ_l *rises*: When discretionary investor d optimally pays attention to shock (or a basket of shocks) l, an increase in its volatility σ_l has a positive impact on expected excess return if $K - \chi(1-\theta)K_{ld} -$

$\underline{\sigma}_{lq}^{-1}\chi\theta > 0$ or equivalently if: $\theta < \frac{K-\chi K_{ld}}{\chi \bar{G}_l}$. The smaller $\bar{G}_l$, the higher the maximum θ for which the condition is satisfied (always satisfied for $\bar{G}_l < 0$).

Increases in $\underline{\sigma}_{i'q}^{-1}$: An increase in the average signal precision of quants about shock $i^{'}$ weakly decreases the attention allocated to it and its marginal benefit, $\lambda_i^{'}$ (prop. 3). Three cases need to be considered. CASE (1): If discretionary investor d did not pay attention to shock $i^{'}$ prior to an increase in $\underline{\sigma}_{i'q}^{-1}$ ($i^{'} = i$), no attention is reallocated (sec. C.2); while the total informational advantage of quants might increase or decrease depending on their total share in the market, $\chi\theta$ (eq. 25). Consequently, d's expected excess return might decrease (low $\chi\theta$) or increase (high $\chi\theta$). CASE (2): If discretionary investor d placed all his attention to asset $i^{'}$ prior to an increase in $\underline{\sigma}_{i'q}^{-1}$, no attention is reallocated (sec. C.2) and an increase in $\underline{\sigma}_{lq}^{-1}$ strictly decreases his expected excess return:

$$\frac{\partial E[R_D - R_M]}{\partial \underline{\sigma}_{lq}^{-1}} = -\frac{1}{\rho}\chi\theta\left[\lambda_l + 2\bar{\sigma}_l\left(\frac{\bar{K}_l}{\rho^2\sigma_x}\left(2\lambda_i - \bar{\sigma}_i\right) + \lambda_l - \bar{\sigma}_l\right)\right]\left(\sigma_{ld}^{-1} - \bar{K}_l\right) < 0 \tag{36}$$

CASE (3): If discretionary investor d paid attention to shock $l^{'}$ prior to the increase in $\underline{\sigma}_{l'q}^{-1}$ as part of a basket of shocks, attention is reallocated until either shock $l^{'}$ is allocated no attention or some of the attention previously allocated to it is diverted to other shocks in the basket (sec. C.2). This always decreases the optimal marginal benefit of learning. Case (3a): When the new basket still contains shock l, the expected excess return of investor d strictly decreases. An increase in $\underline{\sigma}_{l'q}^{-1}$ negatively impacts the portion of portfolio directly involving shock $l^{'}$ (eq. 36), while also lowering the marginal benefit of learning of the other shocks remaining in the basket. Case (3b): When the new basket does not contain shock $l^{'}$, two effects are at play: first, the lower marginal benefit of learning for the shocks remaining in the basket lowers expected excess return; second, the increase in $\underline{\sigma}_{l'q}^{-1}$ might increase or decrease the total benefit of learning of quants, hence decreasing or increasing the expected return of investor d (eq. 25). The net effect depends on the total share of quants in the market ($\chi\theta$). In this scenario $\chi\theta$ would have to be higher than in CASE 1 for the performance of investor d to increase; i.e. the decrease in total benefit of learning of quants would have to more than compensate for the decrease in d's total benefit of learning, due to the reduction in optimal λ. To summarize, an increase in $\underline{\sigma}_{i'q}^{-1}$ for shocks that d pays attention to always decreases his expected excess return. An increase in $\underline{\sigma}_{i'q}^{-1}$ for shocks that d does not pay attention to (or drops from his attention basket) might increase or decrease his expected excess return, depending on $\chi\theta$.

Increases in θ: An increase in θ decreases (resp. increases) the marginal benefit of learning of shocks with a positive (resp. negative) information gap (eq. 26), and it increases the attention allocated by discretionaries to shocks with a lower information gap (sec. C.2). Hence, when discretionaries pay full attention to the aggregate shock, an increase in θ strictly increases their performance: it decrease the marginal benefit of learning of all idiosyncratic shocks (λ_i), decreasing the informational advantage of quants; and it increases the marginal benefit of learning about the aggregate shock (λ_n), increasing the informational advantage of discretionaries. When they pay attention to a basket of shocks which includes the aggregate shock, an increase in θ determines a shift of attention towards the aggregate shock (having it the lowest information gap; $\underline{\sigma}_{nq}^{-1} = 0$), this strictly increases their performance. When they pay attention only to idiosyncratic shocks, an increase in θ strictly increases their performance if these shocks have a negative information gap. If they have a positive information gap, all λs decrease, decreasing both the informational advantage of quants and the total benefit of learning of discretionaries. The net effect depends on the share of quants in the market ($\chi\theta$) and on the magnitude of the information gap.

C.5 *Proposition 6*

The expected excess return of quants is given by:

$$E[R_Q - R_M] = \frac{1}{\rho}\sum_{i\neq l}^{n-1}\left[\lambda_i\left(\underline{\sigma}_{iq}^{-1}(1-\chi\theta)\right)\right] + \frac{1}{\rho}\sum_{l}\lambda_l\left(\underline{\sigma}_{lq}^{-1}(1-\chi\theta) - K_{ld}\chi(1-\theta)\right) - \frac{1}{\rho}\lambda_n K_{nd}$$

l is a basket of idiosyncratic shocks that discretionaries might pay attention to.

Increases in $\boldsymbol{\sigma_{i'}}$ *for* $\boldsymbol{i \in [i, l, n]}$: CASE (1): When discretionaries allocate no attention to the aggregate shock, increases in σ_n or $\sigma_{i\neq l}$ do not lead to attention reallocation (sec. C.4). Hence, increases in σ_n have no effect on the performance of quants, while increases in σ_i for $i \neq l$ strictly increase it ($\frac{\partial\lambda_i}{\partial\sigma_i} > 0$; $\frac{\partial E[R_Q - R_M]}{\partial\sigma_i} > 0$). Increases in $\sigma_{l'}$ *for* $l' \in l$, increase $\lambda_{l'}$ ($\frac{\partial\lambda_l}{\partial\sigma_l} > 0$) as discretionaries reallocate attention towards shock l', leading to a higher optimal marginal benefit of learning about shocks in l. This has a positive impact on the performance of quants if $\underline{\sigma}_{l'q}^{-1}(1-\chi\theta) - K_{l'd}\chi(1-\theta) > 0$, which implies: $\theta < [(\underline{\sigma}_{l'q} - \chi K_{l'd})/(\chi\underline{\sigma}_{l'q} - \chi K_{l'd})]$; this is always verified for a positive information gap ($rhs > 1$; $\theta \in [0,1]$); otherwise, it is verified only if: $K_{l'd} > \underline{\sigma}_{l'q}^{-1} > \chi K_{l'd}$. CASE (2): When discretionaries allocate all attention to the aggregate shock, an increase in σ_i for $i \neq n \neq l$ or in σ_n no attention is reallocated (sec. C.4). Hence, increases in σ_i strictly increase the performance of quants ($\frac{\partial \lambda_i}{\partial \sigma_i} > 0$; $\frac{\partial E[R_Q - R_M]}{\partial \sigma_i} > 0$), while increases in σ_n decrease it ($\frac{\partial \lambda_n}{\partial \sigma_n} > 0$; $\frac{\partial E[R_Q - R_M]}{\partial \sigma_n} < 0$). CASE (3): discretionaries pay attention to a basket of shocks including both the aggregate and idiosyncratic shocks. Case (3a): An increase in the volatility of one of the idiosyncratic shocks in the basket increases the performance of quants, as long as the information gap is sufficiently high (see CASE 1). Case (3b): An increase in the volatility of the aggregate shock, determines a shift of attention to it and away from other shocks in the basket and a net increase in their optimal marginal benefit of learning (sec. C.4). Here we need to consider two cases. (a) When discretionaries still learn about all shocks in the basket, an increase in λ_n strictly increases their performance. To prove this, consider λ^B to be the optimal marginal benefit of learning of all shocks in the basket before the increase in σ_n, and λ^A to be the, strictly greater, optimal marginal benefit after. Without loss of generality assume that discretionaries split their attention between the aggregate shock n and one idiosyncratic shock l. Let's define K_{ld}^B and K_{nd}^B to be the average attention paid by discretionaries to shocks l and n respectively before the increase, and K_{ld}^A and K_{nd}^B to be their average attention after. Finally, recall that discretionaries always allocate the same total capacity (K) to all shocks in the basket, hence $K_{ld}^B + K_{nd}^B = K_{ld}^A + K_{nd}^A = K$. Then, the positive impact on performance due to the decreased attention to shock l ($-\frac{1}{\rho}\chi(1-\theta)(\lambda^A - \lambda_B)(K_{ld}^A - K_{ld}^B) > 0$) is perfectly compensated by the negative impact on performance due to the increased attention to shock n ($-\frac{1}{\rho}\chi(1-\theta)(\lambda^A - \lambda_B)(K_{nd}^A - K_{nd}^B) < 0$). Finally, the strictly higher marginal benefit of learning about shock l positively impacts the total benefit of learning of quants ($\frac{1}{\rho}(1-\chi\theta)(\lambda^A - \lambda_B)\underline{\sigma}_{lq}^{-1} > 0$), increasing their performance. (b) When, following an increase in σ_n discretionaries shift all attention to the aggregate shock, three effects are at play. First, the increases in λ_n and K_{nd} decrease the performance of quants by increasing their informational disadvantage. Second, the lower average signal precision about the shocks dropped from consideration increases their marginal benefit of learning, hence increasing the total benefit of learning of quants and their performance. Finally, an increase in the information gap of the shocks l dropped from consideration increases performance of quants (before: $K_{ld} > 0$, hence $\bar{G}_l = \underline{\sigma}_{l'q}^{-1} - K_{ld} < \underline{\sigma}_{lq}^{-1}$; after $K_{ld} = 0$, hence $\bar{G}_l = \underline{\sigma}_{l'q}^{-1}$). The net effect depends on the share of quants in the market ($\chi\theta$) and on their signal precision ($\underline{\sigma}_{iq}^{-1}$).

Increases in $\boldsymbol{\theta}$: An increase in θ strictly decreases the performance of quants when discretionaries are paying full attention to the aggregate shock: it increases the marginal benefit of learning about the aggregate shock λ_n, increasing the informational advantage of discretionaries; it decreases all other $\lambda_i s$ (they would all have a positive information gap), decreasing the benefit of learning of quants. When discretionaries learn about idiosyncratic shocks with a positive information gap, an increase in θ strictly decreases the performance of quants: it decreases the marginal benefit of learning about all shocks (λ_i). When discretionaries learn about negative information gap idiosyncratic shocks, two effects are at play. First, the marginal benefit of learning about the shocks ignored by discretionaries (λ_i) decreases, decreasing the performance of quants. Second, the marginal benefit of the shocks discretionaries pay attention to (λ_l) increases; this might have a positive impact on the performance of quants, if the gap is not too negative and the total share of skilled investors (χ) is not too high. The net effect, though, is negative as, by definition, these are the shocks to which quants pay the least attention (lowest $\underline{\sigma}_{lq}^{-1}$), hence the negative effect would overweight the potential positive impact. Finally, for a high enough share of quants ($\chi\theta$), increases in $\underline{\sigma}_{iq}^{-1}$ exacerbate the negative impact of increase in θ on λ_i, determining a faster drop in the marginal benefit of learning about those shocks (eq. 27).

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